STATISTICS COMPREHENSIVE EXAM - AUGUST 2012

You have three hours to complete this examination. No references are permitted, but you may use a TI-84 or lower calculator and tables provided by the proctor. Please attempt to **answer all questions**, which are equally weighted. You should present the work you wish to be graded on the scratch sheets provided (one sheet per problem). Partial credit will be given. Please try to present your work as legibly as possible. You may use any lemmas or theorems that you can correctly state (other than one you are trying to prove, of course) to establish your arguments. The problems are not presented in any particular order of difficulty, so you should scan the entire list and begin with the ones that seem the easiest to you. Good luck.

1) State and prove Bayes' Theorem.

2) If X and Y are independent random variables with densities that vanish outside $[0,\alpha]$ and are uniform inside, show that the distribution for $X+Y$ is triangular.

3) Define the hypergeometric distribution, state the conditions under which it is applicable, and derive the formula for it including any restrictions.

4) Consider the function $f(x) = 1 - \frac{x}{2}$ if $x \in [0,2]$ and $f(x) = 0$ otherwise. Show that $f(x)$ is a valid probability density function and compute:

(i) the corresponding cumulative distribution
(ii) the mean
(iii) the variance
(iv) the moment generating function

5) Recall the Cramer-Rao inequality: $var\left(\hat{\theta}\right) \geq \frac{1}{n \cdot E\left[\left(\frac{\partial \ln f(X)}{\partial \theta}\right)^2\right]}$. Show that $\overline{X}$ is a minimum variance unbiased estimator of the mean μ of a normal population.

6) If a sample of size n is drawn from a population with mean μ and standard deviation σ, state the formula for the standard deviation S of the sample and show why S^2 is an unbiased estimator of σ^2.