STATISTICS COMPREHENSIVE EXAM - JANUARY 2013

You have three hours to complete this examination. No references are permitted, but you may use a TI-84 or lower calculator and tables provided by the proctor. Please attempt to **answer all questions**, which are equally weighted. You should present the work you wish to be graded on the scratch sheets provided (one sheet per problem). Partial credit will be given. Please try to present your work as legibly as possible. You may use any lemmas or theorems that you can correctly state (other than one you are trying to prove, of course) to establish your arguments. The problems are not presented in any particular order of difficulty, so you should scan the entire list and begin with the ones that seem the easiest to you. Good luck.

1) State and prove Bayes' Theorem.

2) A cubical die is rolled 120 times with the following result:

$$\begin{bmatrix} face \rightarrow & 1 & 2 & 3 & 4 & 5 & 6 \\ frequency \rightarrow & 18 & 23 & 19 & 24 & 18 & 18 \end{bmatrix}$$

. Present a statistical analysis of why or why not the die should be considered fair (balanced).

3) An urn contains 10 white marbles, 18 red marbles, and 12 blue marbles. Eight marbles are selected from the urn without replacement. What is the probability (at least three decimals) that four are blue?

4) Show that the moment generating function $M_X(t)$ of the normal distribution $n(x;\mu,\sigma)$ is $\exp\left[\mu t + \frac{1}{2}\sigma^2 t^2\right]$.

5) Let s_1 be the standard deviation of a sample of size 16 drawn from a normal population with variance σ_1^2 and s_2 be the standard deviation of a sample of size 16 drawn from a normal population with variance σ_2^2. Assume the samples are independent. Find a 90% confidence interval for estimating $\frac{\sigma_1}{\sigma_2}$.

6) Give a short verbal explanation of the theory behind a one-way ANOVA calculation. You should explain what general issue it is designed to address, the form of typical null and alternative hypotheses, the assumptions made on the random variables involved, how and why total variance is partitioned, and how the appropriate test statistic is constructed.