Instructions:

1. **No references are permitted during the exam.**
2. **Solutions must be written legibly and neatly on separate sheet(s) of paper with your name and problem number at the top of each page.**
3. **Be sure to provide complete and clear reasons for all of your steps (e.g. no statement is "clear" or "obvious" unless it's a definition or given as an assumption, and do not only name a theorem, also show that its hypotheses are satisfied).**
4. **All problems are equally weighted. You have three (3) hours to submit your solutions.**

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1. Two gamblers (say A and B) bet \$1 each on the successive tosses of a fair coin. For each toss, A wins if the outcome is a head and B wins otherwise.
   (a) What is the probability that, after six tosses, A has won \$2 from B ?
   (b) If each gambler has a bank of \$6, what is the probability that A wins all the money from B on the tenth toss?

2. Let X and Y be two random variables with the joint probability density function given by
   $$f(x,y) = \frac{2}{5}(x+4y),\ 0<x<1, 0<y<1.$$
   (a) Derive the marginal distribution of X and the marginal distribution of Y.
   (b) Find $P(X>0.3)$.
   (c) Find $E(Y)$.

3. Let X be the number of tosses of a fair die until a '6' appears.
   (a) What is the probability distribution of X? Provide the probability mass function of X.
   (b) Derive the moment-generating function of the distribution of X.
   (c) Calculate $E(X)$ using the moment-generating function you derived in part (b).

4. Electronic components of a certain type have a length of life X (measured in hours), with density function
   $$f(x) = \frac{1}{100}e^{-x/100}, x>0.$$
   Consider a series system of n such components (operating independently) where the system fails if any one of those components fails.
   (a) For $n=2$, find the probability that such system will last longer than 100 hours .
   (b) Find the expression of median lifetime of such systems for general n.

5. Let $X_1, \ldots, X_n$ constitute a random sample of size n from a continuous uniform distribution over $(0, \beta)$, with β unknown. Consider two estimators of β:

$$\hat{\beta}_1 = 2\overline{X},$$

$$\hat{\beta}_2 = \frac{n+1}{n} Y_n,$$

where $Y_n = max\{X_1, \ldots, X_n\}$. Which estimator do you prefer? Explain.

6. Let $X_1, \ldots, X_n$ constitute a random sample of size n from an exponential population with parameter β. Let θ be the population variance. Find the best unbiased estimator of θ (in terms of $X_1, \ldots, X_n$).