Algebra Comprehensive Exam

Department of Mathematics
Florida Gulf Coast University
Saturday, January 12, 2019

Instructions. You have three hours to complete this examination which consists of **six (6)** equally weighted problems. No references or calculators are permitted. You should present the work you wish to be graded on the scratch sheets provided (one problem per sheet). Partial credit will be given. Please try to present your work as legibly as possible. You may use any lemmas or theorems that you can correctly state, but you must demonstrate their applicability to your arguments. The problems are not presented in any particular order of difficulty, so you should scan the entire list and begin with the ones that seem the easiest to you. Try to allocate your time to maximize points. Good luck!

Problem	Score (out of 10 points)
1	
2	
3	
4	
5	
6	

Name: ______________________________ **Final Score:** ________________

Name: ________________________________

1. State and prove in detail Lagrange's theorem on the possible orders of a subgroup.

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2. (a) State and prove in detail Cayley's Representation Theorem regarding representation of groups as permutations.

   (b) Show in detail the representation of $U(12)$ as a permutation group.

   Remark: As usual, $U(n)$ denotes the set of integers that are relatively prime to n.

Name: ______________________________

3. Prove or disprove: a group of order 56 has a normal subgroup.

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4. Determine the splitting field of $f(x) = x^3 - 2$ over $\mathbb{Q}$. Then determine the Galois group of this splitting field. Draw a diagram showing the correspondence between subgroups of this Galois group and their corresponding fixed fields.

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5. Prove that the ideals (x) and (x, y) are both prime ideals in $\mathbb{Q}[x, y]$ but only (x, y) is maximal.

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6. Prove that every Euclidean domain is a principal ideal domain.