Modern Algebra
Comprehensive Exam Syllabus – August 2018

The algebra comprehensive examination is a three hour test with no references or calculator permitted. There are 6 questions, which are equally weighted, and grading allows partial credit. There has been no advance determination of a passing score. Some of the questions are easy, some more difficult, but there is enough of a range and variety that if you have studied the material below you will be well prepared.

You should be able to state and prove:

(1) Lagrange's Theorem
(2) Tests for subgroups, ideals, submodules
(3) First Isomorphism Theorem for groups
(4) First Isomorphism Theorem for rings
(5) Euclidean domain implies principal ideal domain
(6) Commutative unital ring quotient by prime ideal is domain

You should be able to use:

(1) Second & Third Isomorphism Theorems for groups
(2) Second & Third Isomorphism Theorems for rings
(3) Correspondence Theorem (Lattice) for groups and rings
(4) Gauss' Lemma
(5) Eisenstein's Criterion
(6) Fundamental Theorem for Finite Abelian Groups
(7) Sylow's First, Second, and Third Theorems
(8) Fundamental Theorem of Galois Theory

You should know all the basic definitions concerning:

(1) Types of groups
(2) Major properties of subgroups (normality, etc.)
(3) Commutators and Normalizers
(4) Rings, modules, and fields, including algebraic closure
(5) Reducible and irreducible polynomials over rings and fields
(6) Types of integral domains and their relationships (PID, Euclidean, etc)
(7) Ideals and their properties, such as prime, primary, and maximal ideals
(8) Homomorphisms (injective/surjective) /Isomorphisms/Automorphisms
(9) Galois Field Extensions

You should be able to describe constructions for:

(1) finite fields
(2) direct products
(3) semi-direct products
(4) direct sums
(5) tensor products

You should be familiar with major counterexamples:

(1) integer dividing group order does not imply subgroup
(2) normality is not transitive
(3) not all domains are PID's
(4) an irreducible element is not necessarily prime in some rings

You should be able to state and apply the Fundamental Theorem of Galois Theory.

You should be familiar with field extensions, in general, and be able to prove simple facts about them.