2.7. **Derivatives**

Slope of tangent.

\[
\text{Slope of secant line} = \frac{f(c+h) - f(c)}{c+h - c} = \frac{f(c+h) - f(c)}{h}
\]

Limit of difference quotient.

\[
\text{Slope of tangent at } P = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}
\]

Another version

\[
\text{Slope of tangent at } P = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}
\]

*more suitable if C is known*
\[ \text{Example: Find the slope of tangent at } (2, 5) \text{ of } f(x) = 3x - 1. \]

\[
\text{Slope of tangent} = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \\
= \lim_{h \to 0} \frac{[3(2+h)-1] - [3 \cdot 2 - 1]}{h} \\
= \lim_{h \to 0} \frac{6 + 3h - 1 - 6 + 1}{h} \\
= \lim_{h \to 0} \frac{3}{h} \\
= 3
\]

Slope of tangent at \( x = 2 \) is 3.

\[ \text{Example: Find the slope of tangent line at } t = 5 \text{ of } f(t) = t^3 - 25t - 1. \]

\[
\text{Slope of tangent} = \lim_{h \to 0} \frac{f(t+h) - f(t)}{h} \\
= \lim_{h \to 0} \frac{[(t+h)^3 - 25(t+h) - 1] - [t^3 - 25t - 1]}{h} \\
= \lim_{h \to 0} \frac{t^3 + 3t^2h + 3th^2 + h^3 - 25t - 25h - 1 - t^3 + 25t + 1}{h} \\
= \lim_{h \to 0} \frac{h(3t^2 + 3th + h^2 - 25)}{h} \\
= \lim_{h \to 0} \frac{3t^2 + 3th + h^2 - 25}{h} \\
= 3t^2 + 0 + 0 - 25
\]
Slope of tangent at \( t = 5 \): \( \frac{dy}{dt} = 3(5)^2 - 25 \\
= 75 - 25 \\
= 50 \).

Slope of tangent at \( t = 1 \): \( \frac{dy}{dt} = 3(1)^2 - 25 \\
= -22 \).

2.8 Derivative of a function.

Slope of the tangent at a general point \((x, f(x))\) is called the derivative.

The derivative of \( f \) at \( x \) is given by:

\[
\frac{f(x)}{\text{derivative}} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.
\]

There are few other notations for derivative.

\( f'(x) \), \( \frac{d}{dx} [f(x)] \), \( y' \), \( y'(x) \), \( \frac{dy}{dx} \)
8. Find the derivative of \( f(x) = x^3 + 2x \)

\[
f'(x) = \lim_{{h \to 0}} \frac{f(x+h) - f(x)}{h} \\
= \lim_{{h \to 0}} \frac{[(x+h)^3 + 2(x+h)] - [x^3 + 2x]}{h} \\
= \lim_{{h \to 0}} \frac{x^3 + 3x^2h + 3xh^2 + h^3 + 2x + 2h - x^3 - 2x}{h} \\
= \lim_{{h \to 0}} \frac{h(3x^2 + 3x + h^2 + 2)}{h} \\
= 3x^2 + 2.
\]

8. Find derivative of \( f(x) = x^3 + 2x \) at \( x = 2 \).

\[
f'(2) = 3(2)^2 + 2 = 14.
\]

8. Find equation of tangent of \( f(x) = x^3 + 2x \) at \( x = 2 \).

To find a point on tangent

\[
f(2) = (2)^3 + 2(2) = 12
\]

A point on tangent is \((2, 12)\).

Equation of tangent

\[
y - 12 = m(x - 2) \\
y - 12 = 14(x - 2) \\
y = 14x - 16
\]
Note: If a function $f$ is differentiable at $c$, then it is continuous at $c$.

If a function is continuous at $c \Rightarrow$ function is differentiable at $c$.

\[ f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c} \]

**Left limit**

\[
\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^-} \frac{[-(x-2)] - [0]}{x - 2} = \lim_{x \to 2^-} \frac{-1}{x - 2} = -1
\]

**Right limit**

\[
\lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2} = \lim_{x \to 2^+} \frac{[x - 2] - [0]}{x - 2} = \lim_{x \to 2^+} \frac{1}{x - 2} = 1
\]

Since left and right limits are different \( \lim_{x \to 2} \frac{f(x) - f(2)}{x - 2} \),

\[ \text{DNE, i.e. } f'(2) \text{ DNE} \]
Derivative graph

1. \( f(x) \)
2. \( f'(x) \)