Find inverse function of \( y = 3x + 4 \).

1. \( x \leftrightarrow y \)
   \[ x = 3y + 4 \]
2. Solve for \( y \)
   \[ x - 4 = 3y \]
   \[ \frac{x - 4}{3} = y \]
   \[ y = \frac{x - 4}{3} \]
3. Is this a function? yes
4. replace \( y \) with \( f^{-1}(x) \)
   \[ f^{-1}(x) = \frac{x - 4}{3} \]

**Derivative of inverse function**

Let \( f \) be differentiable function. If \( f \) has an inverse function (i.e. \( f^{-1}(x) = g(x) \)), then \( g \) is differentiable.

\[ g'(x) = \frac{1}{f'(g(x))}, \quad f'(g(x)) \neq 0 \]

Example:

\( f(x) = \frac{x^3}{9} \)

a). What is the value of \( f^{-1}(x) \) at \( x = 24 \)

b). What is the value of \( (f^{-1})'(x) \) at \( x = 24 \)
2. Find \( f'(x) \) at \( x = 24 \).

\[
\begin{align*}
  f(x) &= 24 \\
  \frac{x^3}{q} &= 24 \\
  x^3 &= 216 \\
  x &= \sqrt[3]{216} = 6
\end{align*}
\]

So \( f^{-1}(24) = 6 \).

(b) Find \( f'(f^{-1}(24)) = f'(6) \).

\[
\begin{align*}
  f(x) &= \frac{x^3}{q} \\
  f'(x) &= 3x^2/q = x^2/3 \\
  f'(6) &= 6^{2/3} = 12
\end{align*}
\]

We know
\[
[f^{-1}(x)]' = \frac{1}{f'(f^{-1}(x))}
\]

\[
[f^{-1}(24)]' = \frac{1}{f'(f^{-1}(24))} = \frac{1}{12}.
\]
When a pebble is dropped into a pond, ripples are created in concentric circles. Radius of ripples increases at a rate of 1 ft/sec

When radius is 4 ft, find the rate of change of total area.

Notations
- \( r \) - radius
- \( A \) - Area
- \( t \) - time

Given
\[
\frac{dr}{dt} = 1 \text{ ft/s}
\]

Find
\[
\frac{dA}{dt} = ? \text{ when } r = 4 \text{ ft}
\]

Equation (between \( r \) and \( A \))
\[
A = \pi r^2
\]

Take derivative w.r.t. \( t \)
\[
\frac{dA}{dt} = \pi 2r \cdot \frac{dr}{dt}
\]
\[
= \pi \cdot 2 \cdot (4) \cdot (1)
\]
\[
= 8\pi \text{ ft}^2/\text{sec}
\]
Area is increasing \(8\pi \text{ ft}^2/\text{sec}\).

89. Inflating a balloon. Air is pumped at a rate of \(4.5 \text{ ft}^3/\text{min}\). Find the rate of change of radius when radius is 2 ft.

**Notations**
- \(v\) - volume
- \(t\) - time
- \(r\) - radius

**Given**
\[
\frac{dv}{dt} = 4.5 \text{ ft}^3/\text{min}
\]

**Find**
\[
\frac{dr}{dt} = ? \quad \text{at } r = 2
\]

**Equation**
\[
v = \frac{4}{3} \pi r^3
\]

Find \(\frac{dr}{dt}\)
\[
\frac{dv}{dt} = \frac{4}{3} \pi \cdot 3r^2 \cdot \frac{dr}{dt}
\]

\(\therefore r = 2/\text{ft}\)
\[
\frac{dv}{dt} = \frac{4}{3} \pi 3\left(\frac{2}{\text{ft}}\right)^2
\]

Solve for \(\frac{dr}{dt}\)
\[
\frac{dr}{dt} = \frac{\frac{dv}{dt}}{4\pi r^2}
\]
at \( r = 2 \text{ ft} \)

\[
\frac{dr}{dt} = \frac{1}{4\pi (2)^2} \cdot (4.5) \approx 0.09 \text{ ft/min}.
\]

The radius is increasing at a rate of 0.09 ft per minute.

8. An airplane is flying on a flight path that takes it directly over a radar. If \( S \) is decreasing at a rate of 400 miles/hour when \( S = 10 \) miles, what is the speed of plane at that instant?

Note: \( S \) - distance between radar and plane.

\( t \) - time.

\( x \) - actual distance between A and plane.

Given:

\[
\frac{ds}{dt} = -400 \text{ miles/hour} \quad \text{when} \quad S = 10
\]

Find:

\[
\frac{dx}{dt} = ? \quad \text{at} \quad S = 10 \text{ miles}.
\]
Equation (relation between $s$ and $x$).

$$s^2 = 6^2 + x^2$$

Let us find $x$ value when $s = 10$

$$10^2 = 6^2 + x^2$$
$$x^2 = 100 - 36$$
$$x = \sqrt{64}$$
$$x = 8$$

$$s^2 = 36 + x^2$$

den. value w.r.t. t

$$2s \frac{ds}{dt} = 0 + 2x \frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{s}{x} \frac{ds}{dt}$$

$$= \frac{10}{8}(-400)$$

$$= -500 \text{ miles/hour}$$

Speed of plane is 500 miles/hour.