Using right angle triangle

\[ x^2 = (12-12t)^2 + (6t)^2 \]

\[ x = \sqrt{(12-12t)^2 + (6t)^2} \]

\[ = \sqrt{144t^2 - 288t + 144} \]

Did ships ever sight each other?

To answer this question we need to find the minimum of \( x \).

\[ x = \frac{1}{2}(180t^2 - 288t + 144) \]

\[ = \frac{360t - 288}{2 \sqrt{180t^2 - 288t + 144}} \]

\[ = \frac{72(5t-4)}{2 \sqrt{36(5t^2 - 8t + 4)}} \]

\[ = \frac{72(5t-4)}{12 \sqrt{5t^2 - 8t + 4}} \]

\[ = \frac{6(5t-4)}{\sqrt{5t^2 - 8t + 4}} \]

End of number

\[ \gamma = 0 \]

\[ \frac{6(5t-4)}{\sqrt{5t^2 - 8t + 4}} = 0 \]

\[ 5t-4 = 0 \]

\[ t = 4/5 \text{ hours} \]

Since \( \gamma \) increases as \( t \to \infty \), \( t = 4/5 \) will produce the minimum \( x \) value.

\[ x = \sqrt{180(\frac{4}{5})^2 - 288(\frac{4}{5}) + 144} \]

\[ = \frac{12}{\sqrt{5}} \approx 5.36 \text{ knots} \]

This is the closest distance between 2 ships, and consequently the 2 ships did not sight each other.

Volume of box \( = (50-2x)^2 \cdot x \)

\[ V = x(50-2x)^2 \]

to find maximum volume

\[ V = 4x^3 - 200x^2 + 2500x \]

\[ V' = 12x^2 - 400x + 2500 \]
Conditional number

\[ y^1 = 0 \]
\[ 12x^2 - 400x - 2500 = 0 \]
\[ 4(3x+25)(x-25) = 0 \]
\[ x = \frac{25}{3}, \quad x = 25 \]

Let us find rel max and min

\[ V''(x) = 24x - 400 \]
\[ V''\left(\frac{25}{3}\right) = 24\left(\frac{25}{3}\right) - 400 < 0 \]
\[ V''(25) = 24(25) - 400 > 0 \]

So \( x = \frac{25}{3} \) is the absolute max

Volume at \( x = \frac{25}{3} \) is

\[ V\left(\frac{25}{3}\right) = \left(\frac{25}{3}\right)\left(50 - 2\cdot\frac{25}{3}\right)^2 \]
\[ = \frac{250000}{27} \approx 9259.3 \]

So dimensions are

![Diagram of a box with dimensions](image)

3. \( c(x) = x^3 - 22x^2 + 20000x \)

Average cost

\[ A(x) = \frac{c(x)}{x} \]
\[ = x^2 - 22x + 20000 \]

Critical points of \( A(x) \)

\[ A'(x) = 2x - 22 \]
\[ 0 = 2x - 22 \]
\[ 0 = 2(x-11) \]
\[ x = 11 \]

Show \( x = 11 \) is rel min

\[ A''(11) = 2 > 0 \]

So \( x = 11 \) is the abs min.

4. $7/ft^3$

\[ \$8/ft^4 \]

Note \( xy = 760 \)

So \( y = \frac{760}{x} \)

Cost

\[ c(x,y) = 2x(7) + 2y(8) \]
\[ = 14x + 16y \]

but \( y = \frac{760}{x} \)

So \( c(x) = 14x + 16\cdot\frac{760}{x} \)
\[ = 14x + \frac{12160}{x} \]
to find ordinal points
\[ C'(x) = 14 - \frac{12160}{x^2} \]
\[ 0 = 14 - \frac{12160}{x^2} \]
\[ x^2 = \frac{12160}{14} \approx 868.57 \]
\[ x = 29.47. \]
To show this is rel min
\[ C''(x) = \frac{2(12160)}{x^3} \]
\[ C''(29.47) = \frac{2(12160)}{(29.47)} > 0 \]
So \( x = 29.47 \) is absolute min

So \( y = \frac{760}{x} = \frac{760}{29.47} = 25.79 \)

\[
\begin{array}{c}
25.79 \\
29.47
\end{array}
\]

\[
\begin{array}{c}
\text{5) } \\
x \\
y \\
x \\
y
\end{array}
\]

Find radius of cylinder
\[ 2\pi r = x \]
\[ r = \frac{x}{2\pi} \]

Volume of cylinder
\[ V(x,y) = \pi r^2 h \]
\[ V(x) = \pi \left( \frac{x}{2\pi} \right)^2 (33-2x) \]
\[ = \frac{x^2}{8\pi} (33-2x) \]

to find ordinal points
\[ V(x) = 33x^2 - 2x^3 \]
\[ \frac{\delta}{8\pi} \]
\[ V'(x) = 66x - 6x^2 \]
\[ \frac{\delta}{8\pi} \]
\[ 0 = \frac{6x(11-x)}{8\pi} \]
\[ x = 0, \ x = 11 \]
\[ x = 0 \text{ is not a possible answer.} \]
\[ x = 11 \text{ produce a max} \]
\[ V''(11) = \frac{66 - 12 \times 11}{8\pi} \]
\[ < 0 \]
So \( x = 11 \) is abs max
So \( y = \frac{33-2(11)}{2} = \frac{11}{2} \text{ cm} \)
Earth

\[ q_6 = y + (4x) \]
\[ y = q_6 - 4x \]

Volume

\[ V(x,y) = x^2y \]
\[ V(x) = x^2(q_6 - 4x) \]

Find critical points

\[ V(x) = 96x^2 - 4x^3 \]
\[ V'(x) = 192x - 12x^2 \]
\[ 0 = 192x - 12x^2 \]
\[ 0 = 12x(16 - x) \]
\[ x = 0 \quad \text{and} \quad x = 16 \]

\[ x = 0 \] is not possible.
Show \( x = 16 \) gives a local max

\[ V''(x) = 192 - 24x \]
\[ V''(16) = 192 - 24(16) < 0 \]

So \( x = 16 \) is abs max
So \( y = q_6 - 4(16) = 32 \)

Dimensions are
16 in \times 16 \text{ in} \times 32 \text{ in}

Using similar \( \triangle s \)

\[ \frac{y}{x+30} = \frac{9}{x} \]

\[ y = \frac{9}{x} (x + 30) \]

Using right angle triangle

\[ z^2 = (x+30)^2 + (y)^2 \]
\[ = (x+30)^2 + \left( \frac{9}{x} (x+30) \right)^2 \]

Let \( D = z^2 \). Note when \( D^2 \) is minimum \( z^2 \) will be minimum

\[ D(x) = (x+30)^2 + 81 \left( 1 + \frac{30}{x} \right)^2 \]

\[ D'(x) = 2(x+30) + 162 \left( 1 + \frac{30}{x} \right) \left( -\frac{30}{x^2} \right) = 0 \]

\[ x^3(x+30) - 2430(x+30) = 0 \]
\[ (x^3 - 2430)(x+30) = 0 \]
\[ x = -30 \quad \text{or} \quad x = \sqrt[3]{2430} = 13.444 \]

Note when \( x \to \infty \), \( z \to \infty \)
So \( x = 13.444 \) is the abs min.
Smallest value of $r$

$$z^2 = (x+30)^2 + \left[\frac{9}{x}(x+30)\right]^2$$

$$z = \sqrt{(x+30)^2 + \left[\frac{9}{x}(x+30)\right]^2}$$

$$= \sqrt{(13.444+30)^2 + \left[\frac{9}{1.444}(13.444+30)\right]^2} \approx 52.2802$$
So shortest length occurs when
\[ z = D^2 = (13.276)^2 = 52.2827 \text{ ft} \]

Using right angle triangle.

\[ 10^2 = w^2 + d^2 \]

Stiffness
\[ w = \sqrt{100 - d^2} \]

So
\[ S(w,d) = wd^3 \]

\[ S(d) = \sqrt{100 - d^2} d^3 \]

to find critical points.

\[ S'(d) = \frac{\sqrt{100 - d^2}(3d^2) + d^3 - 2d}{2\sqrt{100 - d^2}} \]

\[ 0 = \sqrt{100 - d^2}(3d^2) - d^4 \]

\[ 0 = (100 - d^2)(3d^2) - d^4 \]

\[ 0 = d^2 [3(100 - d^2) - d^2] \]

\[ 0 = d^2 [300 - 4d^2] \]

\[ d = 0 \text{ or } 4d^2 = 300 \]

\[ d = \pm 8.66 \]

But \( d = 0 \) and \( d = -8.66 \) are not possible

Show \( d = 8.66 \) is a rel max

\[ S'(d) = \frac{4d (3d^4 - 475d^2 + 15000)}{(100 - d^2)^{3/2}} \]

\[ S''(8.66) = \frac{4(8.66)^2 [3(8.66)^2 - 475(8.66) + 15000]}{(100 - 8.66^2)^{3/2}} \]

So \( d = 8.66 \) is abs max

So \( w = \sqrt{100 - (8.66)^2} = 5.0 \text{ in} \)

\[ \text{Fig. 8-7} \]